Math 2085 Linear Algebra
September 5, 2014

1. Consider the following system of linear equations.

$$x + y + z = 2$$
$$y + 2z = 4$$

   (a) Write down the solution set of this system of linear equations using a parameter s for the value of z.

   (b) Now do it again, using a parameter t for the value of y.

   (c) $(0, 0, 2)$ is a solution to this system of linear equations. To what value of s does this correspond? To what value of t? Find another solution of this system and say what values of s and t it corresponds to.

   (d) What does this solution set "look" like?

   (e) Write down three different row echelon forms for this matrix.

   (f) Write down three different reduced row echelon forms for this matrix.

2. (a) If A is a matrix with three rows and five columns, then what is the maximum number of leading 1's in its reduced row echelon form?

   (b) If B is a matrix with three rows and six columns, then what is the maximum number of parameters in the general solution of the linear system with augmented matrix B? Must it have that many parameters? What are the possibilities?

   (c) If C is a matrix with five rows and three columns, then what is the minimum possible number of rows of zeros in any row echelon form of C?

3. Consider the following matrices:

$$A = \begin{pmatrix} 2 & 1 & 5 \\ 3 & 4 & 0 \end{pmatrix}, B = \begin{pmatrix} 2 & 1 \\ 0 & 4 \\ -4 & 2 \end{pmatrix}, C = \begin{pmatrix} 2 & 1 \\ 3 & 4 \end{pmatrix}, D = \begin{pmatrix} 0 & 1 \\ 2 & -1 \end{pmatrix}$$

   (a) Which of these matrices can be added together? What are the results?

   (b) Which of these matrices can be multiplied by -5? What are the results?

   (c) Which of these matrices can be multiplied together?

   (d) Does order matter when you are adding matrices? Is $E + F = F + E$ for every pair of matrices E and F? (Commutativity of addition)

   (e) Does order matter when you are multiplying matrices? Is $EF = FE$ for every pair of matrices E and F? (Commutativity of multiplication)

   (f) Does $E + (F + G) = (E + F) + G$? (Associativity of addition)

   (g) Does $E(FG) = (EF)G$? (Associativity of multiplication)